PAV-89-02

Direct Calculation of Maximum Curvature and Strain in AC Layers of Pavements from Load Deflection Basin Measurements

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Ministry of Transportation

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Direct Calculation of Maximum Curvature and Strain in AC Layers of Pavements from Load Deflection Basin Measurements

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Abstract:	Deflection measurements under impulse load carried out with the Falling Weight Deflectometer (FWD) are usually processed by using elastic layer analysis programs, back-calculating layer moduli from measured deflections, and then forward-computing critical performance parameters such as stresses and strains at locations under the load. In spite of the quality of the programs used, calculations are cumbersome and not always reliable. Presented herein is a more direct, and more reliable way of computing some of those parameters in the top layer, especially the important value of horizontal strain at the bottom of this layer. The measured deflections from a Falling Weight Deflectometer test, the radius of load distribution, and the thickness under the load are the only data needed to calculate the curvature and the strains in the top layer of a pavement structure. It can be shown that strains in the immediate vicinity of the load position, computed with elastic layer analysis methods via back-calculated moduli, are not reliable. In the new proposed method, the maximum curvature of the deflection basin under the load is calculated directly from measured FWD deflections and the radius of load distribution. The strains in the top layer are then determined by the given layer thickness near the load. Corresponding stresses can also be calculated, but additional information is needed, namely the elastic stiffness of the first layer, Poisson's ratio, and the load-induced vertical stress. The new program for calculating curvatures and strains directly from deflection data was tested and could be matched with various, and even extreme cases, computed with elastic layer analysis programs, using their deflection bowl output with closely spaced deflections in the vicinity of the load. The curvatures from the deflection output of elastic layer analysis programs match those calculated by the new proposed method, whereas the strains printed out by these programs were sometimes found to be inconsistent with their own deflection curvature and therefore did not always match with the results of the new program. Evidently, these numerical integration technique programs, in many cases, calculate stresses and strains in the asphaltic layer near the load axis, which are in error due to a state of quasi-singularity imposed on the system by the fairly concentrated wheel load(s). This pertains to an error in underestimating the slope of the strain diagram. An additional discrepancy is caused by the assumption of tensile stresses at the top of the adjacent granular layer. The proposed calculation of strains and stresses (in this order: strains first, then stresses) will contribute and improve the mechanistic phase in the design of asphalt and concrete pavements.
Comments:	The paper was presented in a shorter form, without appendix, to TRB in January 1988, Paper No. 870089
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Project 21172

Ontario

To: Regional Directors
Regional Geotechnical Offices
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Date: 89-8-15

Re: "Direct Calculation of Maximum Curvature and Strain in AC-Layers of Pavements from Load Deflection Measurements"

A short version of this report was first presented and accepted for publication at TRB Annual Meeting, January 1988. The enclosed report contains the full discussion on the topic, including the solution of the differential equation for the circularly loaded AC layer. This solution is the basis for the kinematic or direct way of calculating tensile strains in the AC layer from FWD deflection measurements.

The report is of interest to pavement engineers and researchers who are involved in mechanistic design of pavements using nondestructive testing (NDT) methods for monitoring pavement strength and who want to predict pavement life from standard load tensile strains. The direct calculation of curvature and tensile strain is carried out by a small BASIC program which is now part of a larger program, PROBE, by which raw data from FWD tests can be an analyzed and interpreted.

Thus, the target audience of the report may be rather limited, and outside distribution should be on request only.

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1/ Introduction

To determine the strength and predict the performance of asphalt pavements from some kind of deflection measurement is and has been the subject of major efforts in research for many years. The Falling Weight Deflectometer test (hereafter denoted as FWD) is only one of many methods to measure deflections of pavements; however, it is a very recent one and it is widely accepted in the U.S. It accurately measures a set of deflections of a deflection bowl under an impact load of circular distribution, simulating the transient load of a passing wheel [1, 2]. The method presents a choice of several levels of impact load, corresponding to variations in wheel loads. Deflections are measured at various distances from the load, including at the load axis itself. In operation, the testing equipment is very simple to use, fast and economical.

Traditionally, predictions of pavement performance are made in terms of a number of equivalent 18 kips or 80 kN load applications, usually denoted as N, which causes a decrease in performance and in structural strength. In the case of asphalt pavements, numerical predictions of this number, N, have been derived by experimentally correlating them to one or several parameters of structurally defined critical parameters, such as stresses, strains and/or deflections at certain locations under a standard wheel load of 40 kN. One of these locations is at the bottom of the asphaltic layer underneath the load, where horizontal stresses and/or strains are regarded as critical distress parameters, related to fatigue strength [3, 4].

The state-of-the-art in processing data from FWD measurements is as follows. Asphalt concrete pavements with known layer thicknesses are analysed by using a set of deflections along the wheel path, typically underneath the circular loading plate in the centre and at six adjacent locations, spaced over 1.5 to 2 m. These deflections (together with known layer thicknesses) are used to back-calculate the elastic stiffness (Young's modulus) of all pavement layers, using certain computer programs for elastic layer analysis. Once the elastic stiffnesses are known, stresses, strains, and deflections can be computed by the same or similar compatible programs in forward mode, at any critical location It is also believed that these critical parameters calculated for a standard wheel load, via transfer functions, are related to the service life of the pavement. Ultimately, the purpose is to gain a solid mechanistic basis for the study of such empirical relationships. It is also believed that deterioration of layer materials can be monitored by FWD tests, observing the changes in elastic layer stiffness values.

However, there is an alternative and simpler way to calculate asphalt layer stresses and strains, which can supplement the aforementioned methods. The stresses and strains in any top layer of a

pavement structure can be calculated directly from the measured deflections, based on the theory of elasticity of elastic plates, using the (partial) second derivatives or radii of curvatures, directly obtained from the deflection basin. This basin is given by discrete values from the FWD testing. Through these points of measurement a suitable curve must be fitted.

At first glance this may appear simple, but a certain degree of mathematical accuracy and smoothness is required to obtain reliable and sufficiently accurate second derivatives. This approach is also hampered by a lack of data points in the immediate vicinity of the load axis. The critical maximum curvature is underneath the load, and the difficulty is to find a sufficiently accurate value representing the deflection basin in the centre under the circular loading area. From this curvature (and the given thickness) the strains in the asphalt layer can be calculated without knowing the modulus of elasticity or Poisson's ratio. The thickness underneath the load must be known (but need not be so widely uniform as in the methods of elastic layer analysis).

2/ Stresses and Strains for Single-Tire Wheel and FWD-Test Loads

For single-tire wheels well inside a pavement, the second derivative or curvature of the deflection basin under the load axis can be assumed to be a maximum and equal in all directions because of the circular loading area. For thin plates loaded in this manner the following equations apply:

Plate stiffness $$K = \frac{E\,H^3}{12\,(1-\mu^2)} \quad (1)$$

Bending moment $$M = -K\,W''\,(1+\mu) \quad (2)$$

Horizontal stress $$S = 6\,M / H^2 \quad (3)$$

Horizontal strain $$e = S\,(1-\mu)/E \quad (4)$$

where

- K = bending stiffness of plate
- E = elastic stiffness of asphaltic layer(s)
- H = (total) thickness of asphaltic layer(s)
- μ = Poisson's ratio
- W = deflection as a function of distance
- W'' = second derivative of deflection at zero distance
- M = bending moment under the load, at zero distance
- S = stress at bottom of asphaltic layer at zero distance
- e = strain at bottom of asphaltic layer at zero distance

Note: The second derivative is negative. Bending moments, stresses and strains are equal in both directions.

Without a knowledge of the material properties of the first layer, the modulus, E, or Poisson's ratio, the strain, e, can still be calculated from pure geometrical conditions:

$$\text{either} \quad e = W'' H / 2 \tag{5}$$

$$\text{or} \quad e = H / (2 R) \tag{6}$$

The stress is then:

$$S = e E / (1 - \mu (1 + S_v/S)) \tag{7}$$

where R = radius of curvature in mm
S_v = vertical stress in MPa at or near load

At this stage, in order to calculate corresponding stresses from strains, additional information is required, namely the elastic stiffness (E) of the first asphaltic or concrete layer, the Poisson's ratio of this layer, and the vertical stress (S_v) at the particular point. This vertical component of stress must be obtained by an elastic layer analysis program.

Under a dual wheel the curvature perpendicular to the wheel path may be zero or reversed in sign with a very small value. It can be demonstrated that, in this case, the longitudinal stress is smaller *for the same curvature* in the travel direction by approximately $1/(1 - \mu)$, whereas the longitudinal strain remains the same.

The equations quoted above are based on the concept of regarding the asphaltic layer as an elastic plate. This plate, as a free body, deforms or deflects under two forces; the applied wheel load or falling weight impulse load acting more or less concentrated from above, and the corresponding reactions of soil pressure from the layers underneath, acting onto the plate from below in a more distributed fashion, refer to Figure 1a. For a given load, the deflection bowl, its depth and largeness, depend on the stiffness of the plate and the relative stiffness or strength of the base or soil layers underneath.

3/ Concrete Pavements and Pavement Edges

The calculation of stresses and strains in concrete pavements directly from FWD deflections should be at least as useful as in asphalt pavements. The tensile stresses or strains at the bottom of concrete pavements under a wheel or a falling weight probably constitute an even more potent critical parameter. Moreover, the required input values (aside from the measured deflections), namely the thickness, Young's modulus, and Poisson's ratio, are more definitely determined in concrete slabs than in asphalt layers. Stresses and strains at edges and joints are also important and can be calculated in a similar way, by using curve-fitting with corresponding variations in the underlying assumptions such as free and hinged edges.

4/ Curvature Calculation Inside the Contact Pressure Area

The maximum value of the second derivative W", of the asphaltic layer for circular uniform load (p) is located in the centre of the loaded area. This value, W", can be derived in accordance with elastic plate theory. Within the loaded area, from –a to +a, the tire pressure (p) must be combined with the soil pressure (q) acting from below, resulting in a reduced load diagram (p_x). The resultant load on the plate in this region is assumed to be distributed parabolically, as shown in Figure 2.

The exact solution for this case derived from the differential equation of circular plates is presented in Appendix A.

The result is:

$$W'' = -\frac{2(y_1 - y_a)}{A^2} - \frac{a^2(8p_o + p_a)}{288\,K} \qquad (8)$$

where
- y_1 = maximum deflection in the centre, in mm
- y_a = deflection at the edge of the loaded area, in mm
- a = radius of the loaded area, in mm
- p = circular load (i.e., contact pressure), in MPa
- p_o = resultant pressure in the centre, in MPa
- p_a = resultant pressure at the edge, at a, in MPa
- W" = second derivative in the centre, in 1/mm
- K = plate stiffness, refer to Equation 1

For normal cases of asphalt pavements, the second term constitutes less than 15% of the solution, usually much less.

Without resorting to elastic layer analysis, the contribution of this second term can only be estimated, because p_o and p_a are not generally known. Equation 8 can thus be rewritten as follows:

$$W'' = -\frac{2(y_a - y_a)}{a} - \frac{Cp}{K} \qquad (9)$$

where p = the wheel or FWD impulse load, in kN

C = a coefficient which is defined as follows

$$C = 1000\,(8\,p_o + p_a)/(288\,p\pi) \qquad (10)$$

The coefficient, C, cannot be calculated but, for normal cases, it can be estimated to be somewhere between 4.5 and 5.2. Note that, if C is known, then all values of Equation 9 determined. They are input values from the FWD test, except the deflection y_a, which cannot be measured, but must be calculated by interpolation, using the other measured deflections.

Further, the deflection basin consists of two different parts, i.e., of two curves not continuous in all derivatives. The inner part, from –a to +a, has been accurately derived to obtain Equation 8. The outer part is empirically given by the measurements of the outside sensors, and could therefore be curve-fitted using the method of least squares in conjunction with a Gauss-Jordan algorithm. This curve-fitting becomes more reliable, thus leading to less erratic results, when the centre point deflection is included as part of the curve (as illustrated in Figure 1b and 1c), in spite of the fact that it belongs to the inner part which is mathematically different.

The curve-fitting function chosen is a reciprocal polynomial (Z), and it is used only to calculate the very influential and important value of the deflection, y_a, at the edge of the loaded area, i.e., at $x = a$ by interpolation (Figure 1).

Compared to this approach, the more obvious solution of calculating the second derivative from this overall curve-fitting equation directly (namely by $y'' = 2B/A$) fails to be accurate because the information on the distribution of the load (in the form of the radius, a) is not utilized (refer to Figure 1). This indicates a shortcoming in the curve-fitting equation, and the interpolation process

of computing the deflection, y_a, is still affected by a small systematic error which leads to values of curvature slightly higher than those obtained from using the accurate value of y_a.

Nevertheless, a practical and useful solution has been established which is based on the following discovery:

The first term of Equation 8, using the accurate value of y_a, lead to practically the same curvature as obtained by second differences, for the same data from CHEVRON or ELSYM5 examples.

Comparative computations show that the exact calculation of curvature by Equation 8 results in larger values (up to 15%) than the second differences from CHEVRON or ELSYM5. These results from second differences compare much better with the first term of Equation 8 or 9, omitting the second term. Thus there is a valid argument to dispense with this term which requires additional information about the asphaltic layer, refer also to Appendix A, and to Figure 3. In this case, Equation 11 can be used to calculate the minimum radius of curvature, R.

$$R = -\frac{(y_1 - y_a)^2 + a^2}{2(y_1 - y_a)} \qquad (11)$$

where
- y_1 = maximum deflection in the centre, in mm
- y_a = deflection at the edge of the load disk, in mm
- a = radius of the circular loading area, in mm
- R = radius of curvature, in mm

Note that the first term in the numerator is very small and can be set to zero; then 1/R becomes exactly the first term of the second derivative, W", in Equation 8.

A computer program has been written to calculate curvatures and strains of the asphaltic layer. The program is now a subroutine of the PROBE program, [5].

This subroutine is based on Equation 11, using an estimated value of y_a found by interpolation of a deflection dish function, curve-fitted from relevant FWD measurements (Figure 2)

5/ Example: (Subroutine or Program, based on Equation 11)

The following example constitutes one of the aforementioned verification tests of the proposed method, comparing the subroutine results (based on Equation 11) with that from a corresponding case calculated by the CHEVRON program. For running the new proposed program, calculated CHEVRON deflections are selected as input, simulating feasibly-spaced FWD measurements, in the centre and at four other points outside the loaded area. In addition to these measurements, to run the new program we need:

Radius of load area: a = 150 mm; layer thickness: H = 100 mm

The deflections chosen from the CHEVRON example output, as input into the subroutine:

at	0.0 m	0.4903 mm
at	0.2 m	0.3951 mm
at	0.3 m	0.3394 mm
at	0.4 m	0.2949 mm
at	0.5 m	0.2585 mm

The following values were calculated using the new strain program:

Deflection at the edge of the loaded area: $y_a = 0.4293$ mm
(this is the value calculated by the strain program, the actual CHEVRON value is 0.4305, an inevitable error in curve-fitting)

Radius of curvature at the centre: $R = -184.3$ m
Curvature or second derivative at the centre: $W'' = -5.4255\ (1000/R)$
Strain at the bottom of the asphaltic layer: $e = 271.28\ (\text{mm} \cdot 10^{-6})$

More information was needed to run the CHEVRON program, namely:

Load: 40.01 kN, tire pressure: 566.00 kPa, load radius: 150 mm
Layer values:
1) Modulus: 3000 MPa, Poisson's ratio: 0.35, Thickness: 100 mm
2) Modulus: 500.MPa, Poisson's ratio: 0.35, Thickness: 200 mm
3) Modulus: 80 MPa, Poisson's ratio: 0.35, Thickness: infinite

Note: 1) = surface course, 2) = base course, 3) = subgrade.

Using these input values for the CHEVRON program, the deflections were calculated every 50 mm. Then the finite difference method was used to compute the second derivatives between X = 0 and 150 mm manually.

The results are somewhat "wobbly", and the average was found to be –5.37, with a slightly smaller value at X = 0 of –5.28. This compares fairly well with the computed value of –5.4255 of the output listed above.

Figure 4 shows a comparison of the corresponding strain calculations. The strain diagram from the direct CHEVRON printout is slightly curved at the top; it also exhibits a small net compressive strain. The strains of the direct output ($+185.7 \cdot 10^{-6}$ and $-246 \cdot 10^{-6}$) are smaller, especially at the bottom, and the steeper slope of this strain diagram seems to be inconsistent with the second derivative, manually computed from the same output (–5.28).

With the strain calculated in the printout above, the stress at the bottom of the asphalt layer can be computed by means of Equation 7. Setting $S_v = 0$, the result would be:

$$S = 0.0002713 \cdot 3000/0.65 = 1.30 \text{ MPa (tensile stress)}$$

The vertical stress, S, under the load is not zero but rather is negative, a fraction of the vertical contact pressure. Let us estimate the ratio of S_v/S to be –0.5; the value μ is then reduced to $0.5 \cdot 0.35 = 0.175$, and the factor $e \cdot E$ must be divided by 0.825 instead of 0.65.

The result is:

$$S = 0.0002713 \cdot 3000/0.825 = 0.9865 \text{ MPa}$$

The corresponding value printed out from CHEVRON is S = 0.6975 only, which is obviously too low.

The compressive stresses on the top, however, are closer:

+ 1.252 MPa ($S_v = 0$) versus + 1.44 MPa of the CHEVRON printout.

The example illustrates that curvatures or second derivatives (1/R or W") can be calculated with great confidence and sufficient accuracy. The strain computed either as $e = W'' \cdot H/2$, or as $e = H / 2R$ is certainly more reliable than the inconsistent printout of the CHEVRON program. The stress, on the other hand, is dependent not only on layer material constants (E and μ), but also on the vertical stress component, which , in the vicinity of the applied load, can be significant.

Table 1

Comparison of the Strain Program with CHEVRON/ELSYM5

Description	Curvatures		Strain		
	$\cdot 10^{-6}$ 1/mm		$\cdot 10^{-6}$		
	DIRECT	ELSYM5/ CHEVRON	DIRECT	ELSYM5/CHEVRON	
				bottom	average
Typical 3-layer case a = 150 mm, H = 100 mm	5.160	–4.80	258.0	170.4	193
Typical 4-layer case a = 150 mm, H = 100 mm	–5.426	–5.28	271.3	185.7	255
Overload case, 2.5x a = 150 mm, H = 100 mm	–14.46	–13.8	642.6	423.0	520
Deep strength 140 mm on weak soil	–4.334	–4.53	302.7	278.2	278
Reduced area a = 102 mm high pressure 400 psi	–26.97	–26.75	1198.9	786.4	860
Wider area, a = 203 mm normal pressure	–8.753	–8.66	389.0	243.2	341
Typical 3-layer case normal pressure	–5.066	–5.18	253.3	208.2	204
Very soft layer inserted under asphalt	–8.438	–8.74	421.9	383.8	385

Note: In the last column, "average" means the average of the absolute value of the top and bottom strains of the first layer from the output listing of CHEVRON or ELSYM5. These values in the last column ought to be close to or equal to H/2 times the curvature values in the second column. This identity check would establish consistency within the CHEVRON or ELSYM5 programs themselves, not involving the new strain model. Note that the check fails except in the case of a very soft inserted layer in the last line.

6/ Testing of the Strain Program

The new program for computing strain has been tested against cases calculated by elastic layer analysis programs (CHEVRON and ELSYM5), using densely spaced deflection points underneath the area of load distribution. The curvatures were calculated from these deflections via second differences (refer to second column in Table 1) and the strains were taken from the printed output (last two columns).

Then, five deflections were chosen from the same printouts, one at the centre and four outside the loaded area, pretending these to be feasible FWD readings. They were used as input into the Strain program, together with the radius, a, and the thickness, H. The results, the curvatures and horizontal strains at the bottom of the asphaltic layer, are listed under Strain in Table 1, for a variety of cases, including extreme ones. The curvatures agree fairly well, whereas the strains from the elastic layer analysis programs are shown to be inconsistent with their own curvatures.

The ELSYM5 program was used to carry out further tests over a wider range of asphalt layer thicknesses, this time using not only the top but also top/bottom-average deflections. The following observations were made:

Equation 11, with accurate values of y_a, works slightly better with average deflections (top/bottom) than with top deflections. the differences, however, are insignificant.

Secondly, for the region under the distributed load, from –a to +a, the curvature or second derivative has been independently derived by solving the differential equation of elastic circular plates, resulting in Equation 8, or Equations 9 and 10. Using these equations, we can calculate the curvatures and compare them with corresponding results from using the second differences of the deflections printed out by CHEVRON or ELSYM5. Eight cases have been calculated and compared in this way, as in Table 1. The results are listed in Table 2. The comparison exhibits differences in the same direction as before, mostly somewhat larger.

Table 2

Curvature by Second Differences and Equation 8/9
(times 10)

CASE (see also Table 1)	SECOND DIFFERENCES	EQUATION 9/10
1	–4.80	–5.475
2	–5.28/–5.68 (wobbling)	–6.018
3	–13.78	–15.535
4	–4.53	–4.603
5	–26.75	–28.931
6	–8.66	–9.722
7	–5.182	–5.562
8	–8.744	–8.875

The curvatures calculated by Equations 8/9 are slightly higher than the ones computed from the ELSYM5 deflections around the load.

7/ Conclusions and Recommendations

Using the Strain program, the curvature of the asphaltic layer and the horizontal strain at the bottom of this layer can be calculated directly from the input of the measured values of the deflection basin, the load distribution radius, and the thickness of the asphaltic layer at the load position. Because no other information is needed (about layer materials and thicknesses of lower soil layers), the horizontal strain calculated in this particular way is a very potent parameter for pavement design.

The bottom strain in the asphalt layer so calculated is larger than the strain from elastic layer analysis in which full friction between asphaltic and granular layers is assumed, which leads to tensile stresses at the top of the granular base.

Contrary to the proposed calculation of strain, the corresponding horizontal stresses are affected by the usual uncertainties of determining material parameters, elastic stiffness, and Poisson's ratio, and by variations of layer thickness.

The direct calculation of curvature and strain by the new program has been successfully tested against more detailed computations and output from CHEVRON or ELSYM5 programs. The second derivatives of the deflections, the curvatures, agree fairly well, but the corresponding printed-out strains were found to be inconsistent with CHEVRON's or ELSYM5's own deflections. Tensile strains under the load in the bottom fibres of asphaltic concrete layers calculated by elastic layers analysis programs (CHEVRON, ELSYM5) are too small for two reasons:

First, there is error of slope. The slope of the strain diagram is too small because of quasi-singularity from the fairly concentrated load (Figure 4).

Secondly, the assumption of shear resistance between the asphaltic layer and the next lower layer of granular material leads to apparent tensile stresses at the top of this granular base layer. This "pulls down" the neutral axis of the strain diagram in the top asphaltic layer (Figure 4).

The new model has also been compared with an independently derived formula, based on the solution of the differential equation of elastic plates for the region under the circular load. The variation of curvature within the loaded area has been studied and discusses: CHEVRON and ELSYM5 deflection printouts, via second differences, seem to closely approximate an average curvature between the maximum value at the load axis and the much lower value at a distance from the axis equal to the layer thickness. This average curvature is slightly smaller than the maximum curvature in the centre, but sufficiently close to it (Appendix A, and Figure 4).

Since the strain parameter is generally conceived to be correlated to the fatigue strength of asphaltic pavement layers, it is recommended to routinely process data from Falling Weight Deflectometer tests by this new strain criterion, in addition and supplementary to other current FWD processing methods.

The strain is a geometrically calculated elongation of horizontal layer fibres per unit length, directly and linearly proportional to curvature and thickness of the layer. It is related to stress in the usual way only as long as the asphaltic layer is new and unharmed, and no cracks have initiated at its bottom. Therefore, strain can be regarded as a parameter to monitor the deterioration of the asphaltic layer.

This means that, in the course of time, periodic FWD tests would exhibit an excessive increase in strain and curvature. (In contrast to this, the deterioration of subgrade and soil layers would lead to a general increase in deflections across the whole deflection basin.)

The "kinematic" method of direct calculation of AC strain described in this report has been incorporated into the author's PROBE program [5].

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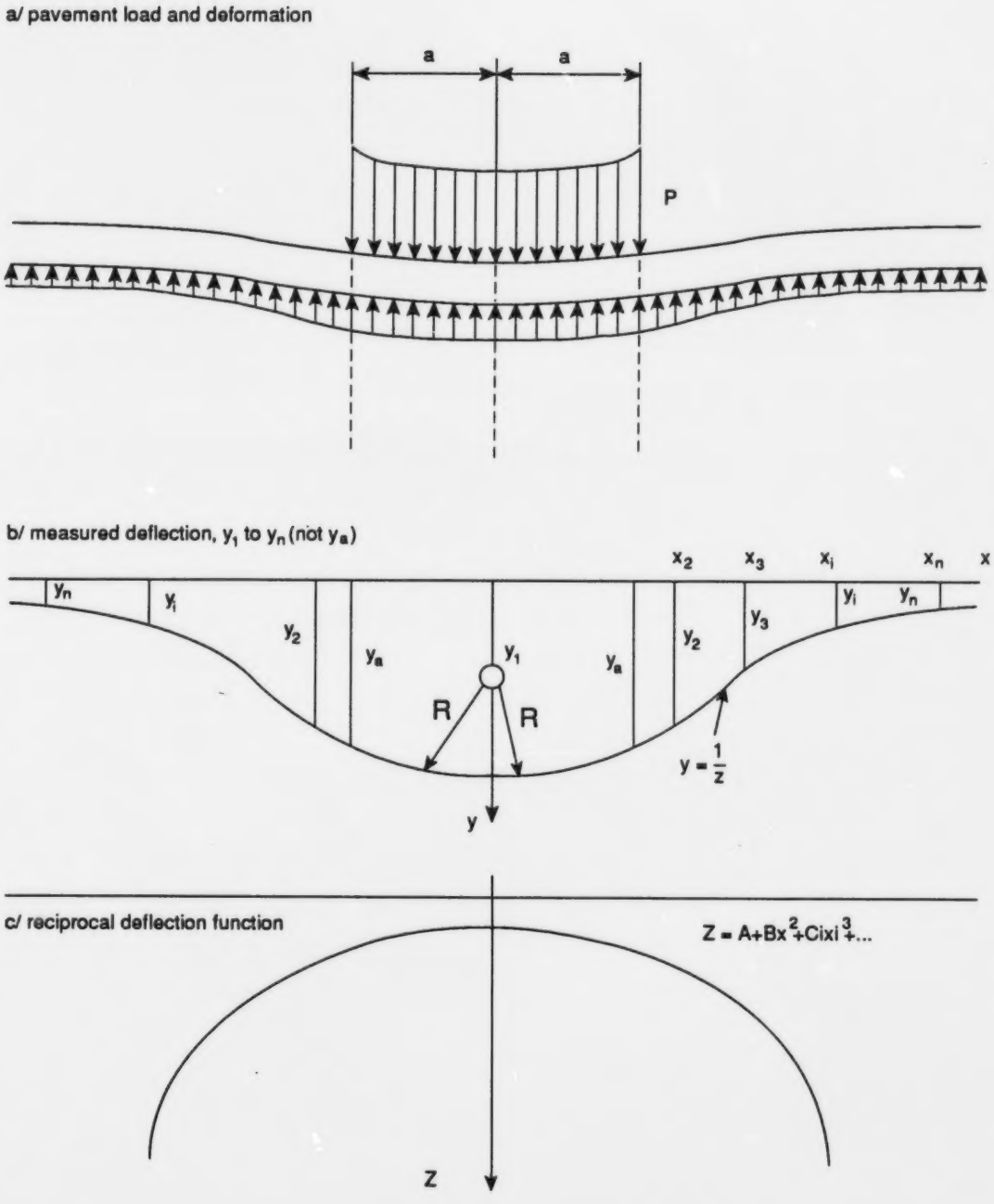

Figure 1/ Deflection Basin Geometry

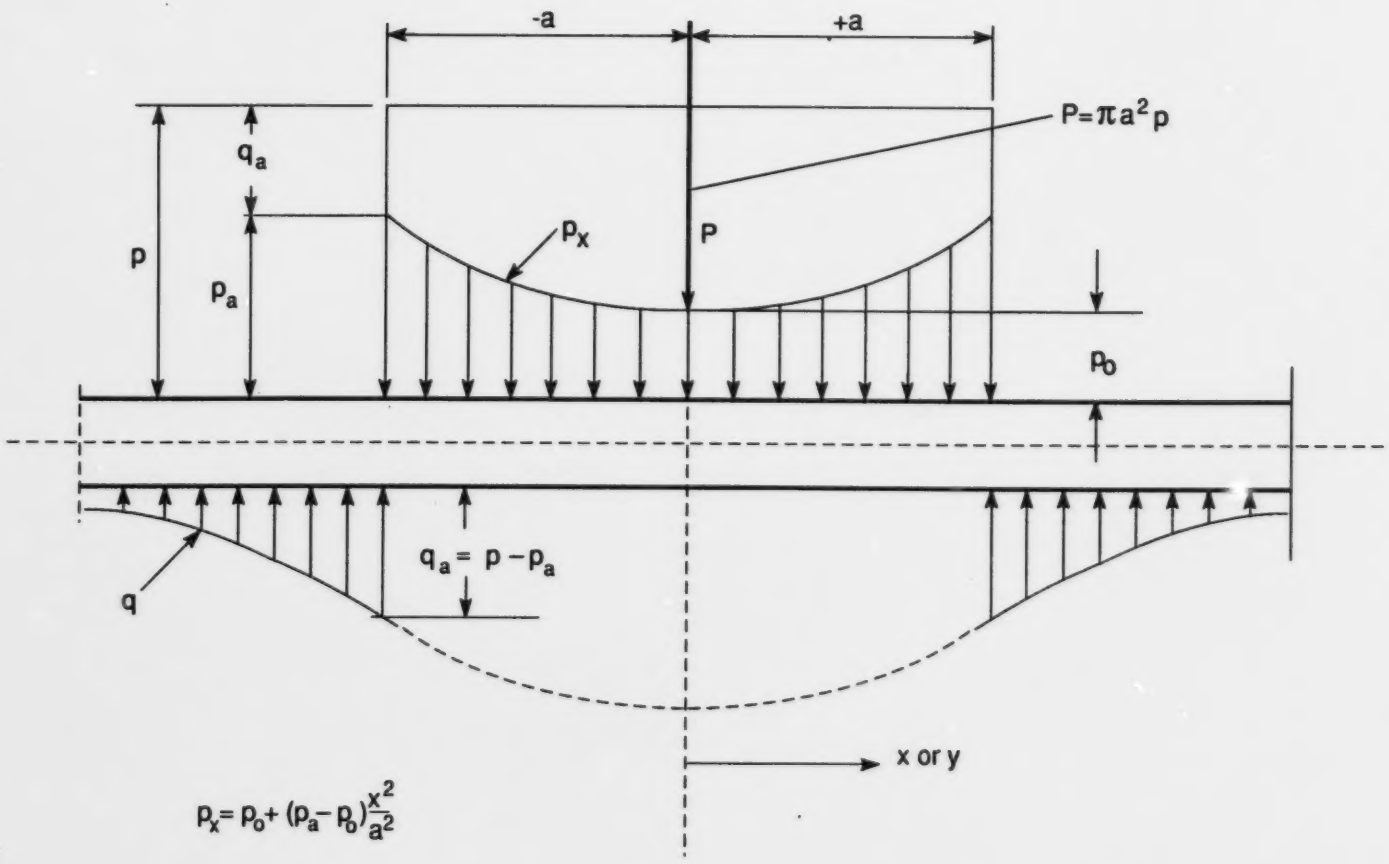

Figure 2/ Load Distribution Between –a and +a

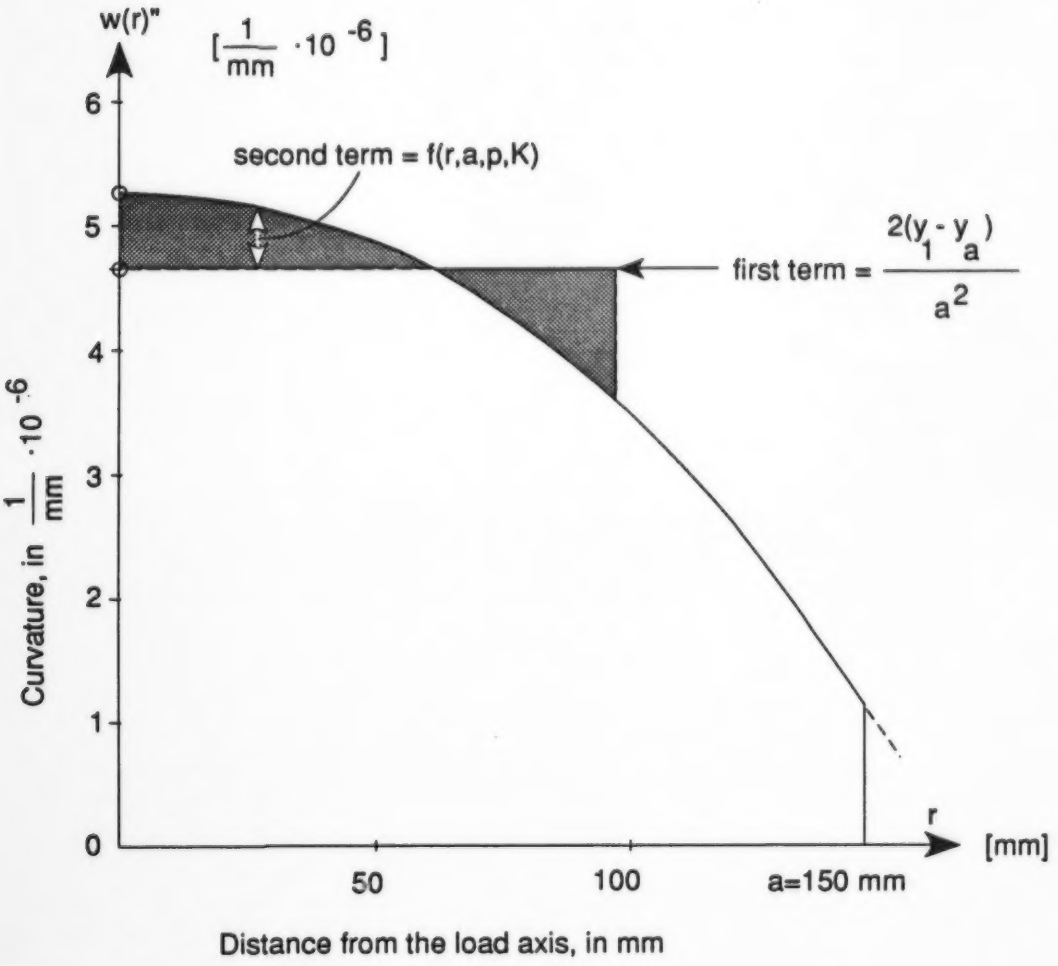

Figure 3/ Radial Curvature in the Asphaltic Layer

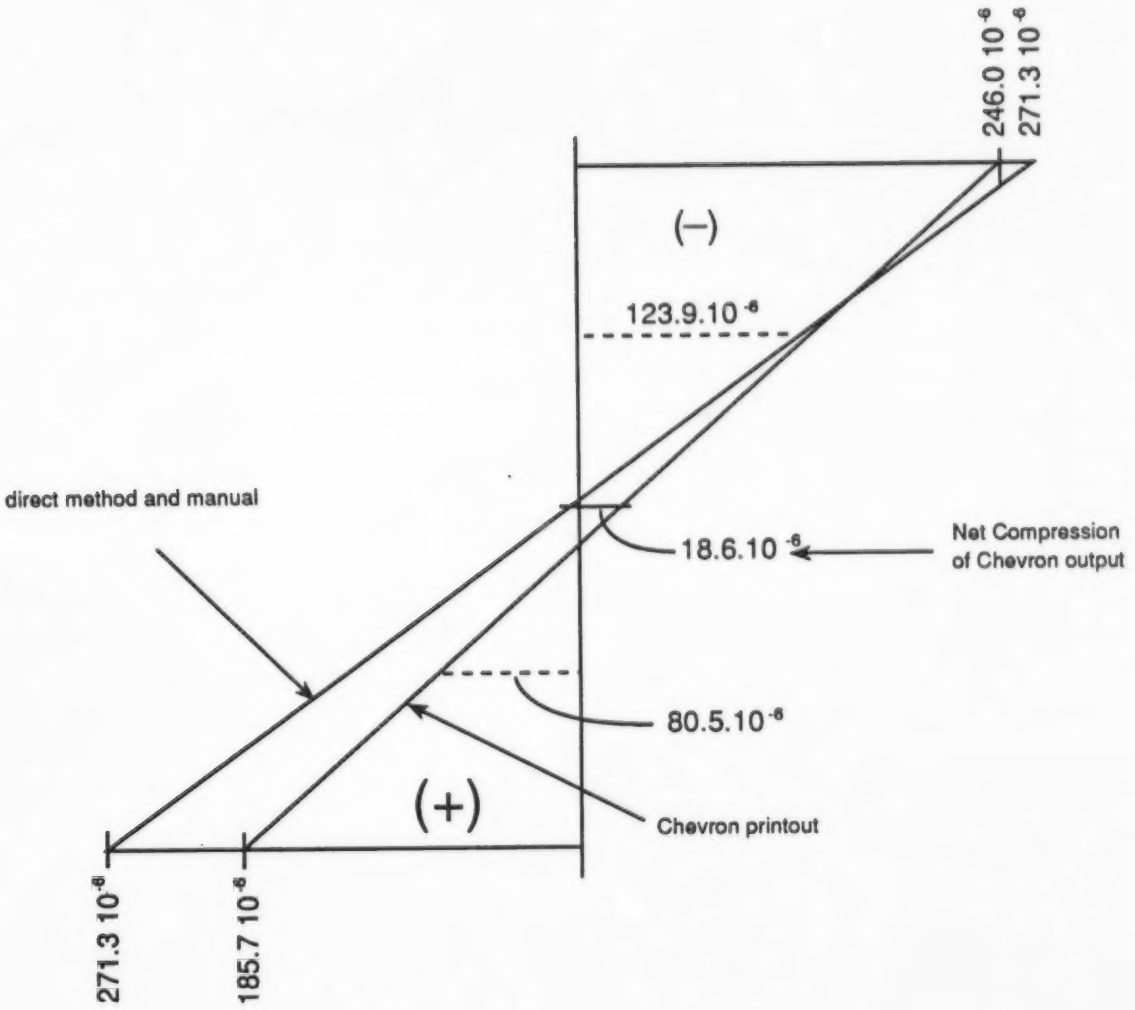

Figure 4/ Comparison of Strain Calculation (Chevron Example)

APPENDIX A

Derivation of Curvature from Differential Equation of Elastic Circular Plates within Loaded Area, a

From the point of view of the theory of elastic plates, concentrated loads are points of singularity. For instance, influence surfaces for bending moments at the point of moment are drawn out into an infinitely long, narrow hose, so that stresses and strains at this point from a concentrated load at the same point become somewhat uncertain. Heavy wheel loads distributed over a small circular area are still affected by such uncertainties and inaccuracies due to a state of quasi-singularity.

This is the reason we find inconsistencies in elastic layer analysis program calculations with CHEVRON or ELSYM5, which are as affected by this problem of close or quasi-singularity as corresponding cases calculated by the theory of elastic plates. Moreover, in all cases of full or quasi-singularity, the assumptions about the the distribution of the load as expressed by the load radius, a, become very critical.

Thus it is reasonable to pursue a classical approach, namely to derive curvatures and strains under the load axis based on the theory of elastic plates -- we know that the results must be true for thin plates, i.e., for cases of layer thicknesses, H, being smaller than the load radius, a.

With reference to Figure 2, the asphaltic layer is regarded as an elastic plate with uniform thickness, H. The pavement surface is loaded by a circular uniformly distributed load, p (MPa). However, this is not the load of the plate. The vertical upward pressure beneath the plate must be deducted. This is p_o in the centre and p_a at the load radius, a.

More specifically:

Load per unit area in the centre: $$p_o = p - q_o \qquad \text{(A-1)}$$

Load per unit area at the edge, a: $$p_a = p - q_a \qquad \text{(A-2)}$$

Assuming a parabolic distribution, we can write:

$$p_r = p_o + (p_a - p_o)\,(r^2/a^2) \qquad \text{(A-3)}$$

Denote: a = load radius, mm
r = radius or distance from centre of load, mm
q_o = upward soil pressure at the bottom of the asphaltic layer in the centre, in MPa
q_a = the same, at r = a, in MPa
p = tire pressure or disc pressure, in MPa
p_o = resultant area load at r = 0, in MPa
p_a = resultant area load at r = a, in MPa
p_r = vertical area load as function of r, in MPa
H = thickness of asphaltic layer, in mm
E = modulus of elasticity of asphaltic layer, in MPa
μ = Poisson's ratio

The general form of the solution of the differential equation of bending for circular plates can be written as follows:

Plate stiffness: $$K = \frac{E H^3}{12 (1 - \mu^2)} \quad \text{(A-4)}$$

Particular solution: $$w_o = - \frac{1}{k} \int \frac{dr}{r} \int M r \, dr \quad \text{(A-5)}$$

Sum of moments: $$M = - \int \frac{dr}{r} \int p_r r \, dr \quad \text{(A-6)}$$

General solution: $$w = w_o + C_1 + C_2 r^2 \quad \text{(A-7)}$$

where: w = deflection as a function of r, in mm
w_o = particular solution for deflection as a function of r, in mm
C_2, C_1 = integration constants

Substituting Equation A-3 for p_r in Equation A-6, we can solve M by double integration:

$$M = - [\frac{p_o r^2}{4} + \frac{(p_a - p_o) r^4}{16 a^2}] \quad \text{(A-8)}$$

Substituting Equation A-8 for M in Equation A-5, we can find the particular solution, w, by double integration:

$$w_o = \frac{1}{k}\left[\frac{p_o r^4}{64} + \frac{(p_a - p_o) r^6}{576 a^2}\right] \qquad \text{(A-9)}$$

Equation A-9, and consideration of boundary conditions, leads to an actual solution of Equation A-7.

Boundary Conditions

1) For r = 0: $w = 0$ and $C_2 r^2 = 0$,
therefore: $C_1 = y_1$ = maximum deflection in the centre, in mm

2) For r = a: $w = y_a$ = deflection at the edge of the loaded area, in mm

This leads to an expression for $C_2 r^2$.

Finally, the equation for the deflection, w, which is the solution of Equation A-7, is:

$$w(r) = -(y_1 - y_a)\frac{r^2}{a^2} - \frac{a^2 r^2}{64 K}\left[p_o + \frac{1}{9}(p_a - p_o)\right]$$

$$= y_1 \frac{1}{64 K}\left[p_o r^4 + \frac{1}{9}(p_a - p_o)\frac{r^6}{a^2}\right] \qquad \text{(A-10)}$$

The curvature of the deflection line is the second derivative of Equation A-10 with respect to r:

$$w''(r) = -\frac{2(y_1 - y_a)}{a^2} - \frac{p_o}{32 K}(a^2 - 6r^2) - \frac{p_a - p_o}{288 K a^2}(a^4 - 15 r^4) \qquad \text{(A-11)}$$

Equation A-11 has one major term independent from the load (p_a ,p_o) and two terms which are a function of the resultant load. Note that from r = 0 to r = a the load-dependent terms change sign at about r = 0.408 a and r = 0.508 a, respectively. The largest (absolute) value of curvature is at r = 0, and can be written as:

$$w''(r) = -\frac{2(y_1 - y_a)}{a^2} - \frac{a^2}{288\,K}(8p_o + p_a) \qquad \text{(A-12)}$$

If the area loads p and pa were zero, i.e., if the tire load was mostly distributed near the radius, a, except for a small amount counteracting q (and q_a), then there would be a constant curvature from r = 0 to r = a, namely $-2(y - y_a)/a^2$. Indeed, it can be shown.that this first term is predominant in all normal cases and that the second term is less than 15% of the total w"(0).

Finally, looking at both equations (A-11 and A-12), we may say that the curvature is predominantly the first part $[-2(y_1 - y_a)/a^2]$, modified by minor disturbances due to load distribution. A numerical example will clarify this further.

Example (Equation A-11)

y_1 = 0.4093 mm
y_a = 0.3557 mm
a = 150 mm
E = 3000 MPa
μ = 0.350
H = 100 mm

Printout from a CHEVRON example

p = 0.5660 MPa
q_o = 0.3157 MPa
q_a = 0.1699 MPa

$$p_o = 0.5660 - 0.3157 = 0.2503 \text{ MPa} \qquad \text{(A-1)}$$

$$p_a = 0.5660 - 0.1699 = 0.3961 \text{ MPa} \qquad \text{(A-2)}$$

$$K = \frac{3000.100^3}{12\,(1 + 0.35^2)} = 284.9 \times 10^6 \qquad \text{(A-4)}$$

$$w(r)'' = -\frac{2(y_1 - y_a)}{a^2} - \frac{p_o a^2}{32\,K}\left[1 - 6\left(\frac{r}{a}\right)^2\right]$$

$$- \frac{(p_a - p_o)\,a^2}{288\,K}\left[1 - 15\left(\frac{r}{a}\right)^4\right] \qquad \text{(A-11)}$$

Equation A-11 has been rewritten to render it a function of S = r/a:

$$w(r)'' = -[4.764 + 0.618\,(1 - 6\,S^2) + 0.040 \bullet (1 - 15S^4)] \times 10^6$$

This function has been plotted as Figure 3.

Figure 3 illustrates, for a normal case (line 1 in Tables 1 and 2), the function of radial curvature in the asphaltic layer, from the centre (r = 0) to the edge of the loaded area (r = a). Let us suppose that we were to look for an average curvature between r = 0 and r = H (<a), then we find that the first term of Equation A-11 is a close approximation (4.764×10^{-6} 1/mm). This value of the first term agrees with the curvature obtained by second differences of CHEVRON output (namely, 4.8×10^{-6} 1/mm).

If there is no horizontal tension in the granular base layer underneath the asphalt (how could there be?), then the strain at the bottom of the asphaltic layer cannot be smaller than:

$$\varepsilon = H \bullet (y_1 - y_a)/a^2 \qquad \text{(A-13)}$$

That is, the first term of Equation A-11 times half the layer thickness (H). In our example:

$$\varepsilon = 100 \bullet (0.4093 - 0.3557)/150^2$$

$$= 238.2 \bullet 10^{-6}$$